ALGEBRA for All

James R. Choike

Teaching Strategies for "Algebra for All"

Educators and policymakers alike recognize that algebra is an important gatekeeper course, not only for college preparation but also for preparation for the world of work. To prepare students for future success, many school districts and state legislatures now make algebra a graduation requirement for all high school students. Of course, the burden for preparing all students in algebra falls on the shoulders of the classroom teacher. Whether an algebra-for-all initiative is school-, district-, or state-based, a teacher faces the difficult challenge of teaching a high-standards course to a classroom of students whose beginning knowledge may range from far below to far above the course prerequisites.

Since 1990 I have been actively involved in the College Board's algebra-and-geometry-for-all school-reform project called *Equity 2000*. Equity 2000 is an educational reform initiative for grades K–12 that is based on the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). It uses mathematics—algebra and geometry by the tenth grade—as a lever to enhance academic achievement for all students, especially minority students, thereby increasing the likelihood that they will enroll in and complete college. My work for Equity 2000 consists of serving as chair of the Equity 2000 National Mathematics Technical Assistance Committee and as an in-service presenter at Equity 2000 professional development institutes for mathematics teachers. This article shares some lessons that I have learned from my Equity 2000 experiences.

Focus instruction on a few big ideas

The article presents classroom strategies that help me in my own teaching at the university level and in in-service presentations. These strategies have evolved over time. They have been the result of a gradual personal professional development stemming from the challenges that I have faced in being asked by the College Board to lead Equity 2000 teachers by offering insights and strategies to help them reach all students. They have also been formed from working with teachers and students in Equity 2000. In this leadership role, I have worked toward preparing algebra-for-all exemplar classroom materials for Equity 2000 in-service presentations to teachers and have demonstrated these lessons for teachers by modeling their use and pedagogy with actual classes of students that are typical of those that they are likely to face within their districts.

For example, in 1992, I teamed with J. T. Sutcliffe, a mathematics teacher at Saint Mark's School of Texas, to create and demonstrate five algebra-for-all lessons with a group of thirty-two eighth graders from Prince George's County, Maryland. Subsequently, in 1993, I teamed with Rheta Rubenstein of Schoolcraft College, in Livonia, Michigan, in creating and demonstrating lessons in geometry with thirty tenth-grade students in an Equity 2000 Summer Math Institute in Milwaukee.

Each algebra-for-all teaching strategy includes a brief explanation of the nature and scope of the suggested strategy. Although each one has general application to teaching mathematics, the strategies are presented in the setting of teaching algebra.

Emphasize conceptual understanding, that is, emphasize "big ideas"

A daunting task that teachers face throughout the school year is "getting through the book." The table of contents of most algebra textbooks compartmentalizes algebra into many chapters and many topics within each chapter. But algebra essentially involves just a few conceptual themes, or "big ideas." Focusing instruction on a few big ideas accomplishes two important instructional goals: (1) it helps *all* students understand how a topic that they are currently learning connects with material previously learned; and (2) it gives a conceptual framework for teaching a high-standards course without being bound to follow the textbook one section after another. Selected big ideas are illustrated throughout the rest of this article.

Jim Choike, choike@math.okstate.edu, teaches at Oklahoma State University, Stillwater, OK 74078. He is interested in pedagogy, curriculum development, technology, and professional development for teachers in algebra, geometry, Advanced Placement calculus, and collegiate mathematics.

Eliminate numbers as distracters to understanding new concepts

Consider the following problem.

> A department store has a Surround Sound/CD player system on sale for a down payment of \$13.80 and a monthly payment of \$22.69.
>
> *a*) Write an algebraic expression that allows you to compute the amount paid after any payment number.
>
> *b*) How much will you have paid for the CD system after the twenty-fourth payment?

This problem has a lot to offer algebra students. Unfortunately, the reality-based amounts of \$13.80 for the down payment and \$22.69 for the monthly payment may be more than some students' existing skills can handle. Such numbers as 13.80 and 22.69 can actually distract students from understanding simple concepts that underlie the solution of a problem. As a preliminary step to solving the problem, eliminate the decimals. For example, have students solve a similar problem in which the down payment is \$5 and the monthly payment is \$10. With these "friendly" numbers, more students are likely to build a table of cost in terms of payment number and to recognize the pattern in this table both in words and as an algebraic expression, as requested in part (*a*) of the problem. This preliminary problem also offers students insights about how they can solve the original problem.

Ensure entry into a problem or activity

In other words, questions of clarification from your students should always be permitted. Consider the following problem, which appeared as the first problem in *Addison-Wesley Mathematics: Grade 6* (Eicholz et al. 1991) in a section entitled "Using Whole Number Division to Solve Problems."

> A golden eagle takes about 56 hours to hatch. It may take the chick 3 times as long to break out of the shell as to peck the first hole. How long does it take the chick to peck the first hole?

This problem includes three ambiguous elements that are not inherently mathematical, that confuse students, and that prevent them from getting started on, that is, entering, the problem solution. These elements are as follows:

- Eagle versus chick: students may be unclear about the relationship between the golden eagle and a baby chick. If the problem statement had used the words *baby eagle* instead of *chick,* this relationship would then be clear to students.
- The use of the words *about,* as in "about 56 hours to hatch," and *may,* in "may take the chick," introduce an element of uncertainty to some students about the numbers to use in this problem. Compare these wordings with the wording in the question statement of the problem, which very definitely asks, "How long does it take the chick to peck the first hole?"
- The meaning of *hatching* is somewhat ambiguous. Does hatching begin when the chick starts to peck the hole (this interpretation leads to an equation $x + 3x = 56$) or when the first sign of a crack appears to an outside observer (this interpretation leads to the equation $3x = 56$)?

Clarification questions should always be encouraged

The golden-eagle problem is an example of a problem in which a student's inability to solve it may have no direct connection with his or her level of mathematical knowledge. Both the strong student and the weak student may have trouble with this problem. Providing entry to a mathematical activity or a problem is a strategy that ensures that *all* students understand the setting of a problem.

Emphasize multiple representation: words, tables of data, graphs, and symbols

Multiple representation is a content "big idea" of algebra. Students should be taught the value of representing mathematics verbally in words, numerically in tables, visually in graphs, and algebraically in symbols and should learn how these various representational forms of mathematics are connected.

Multiple representations also help reach the goal of algebra for all. A teaching strategy that connects the various forms of multiple representations to describe mathematics is an effective strategy for reaching out to students with different learning styles. In this sense, multiple representation provides an algebra-for-all pedagogy strategy in the classroom.

Multiple representation is also a problem-solving strategy. Using the full variety and power of all the forms of describing and presenting mathematics is an effective means for gaining insight into, and understanding of, a problem situation. In this sense, multiple representation is a problem-solving process.

As an example of the power of multiple representation, consider the following problem:

> List the ways to change a fifty-dollar bill into five-dollar and/or twenty-dollar bills.

A straightforward, numerical trial-and-error approach by students yields the following three ways to change a fifty-dollar bill: (1) two 5s and two 20s, (2) six 5s and one 20, and (3) ten 5s and zero 20s.

By placing the algebraic equation $5x + 20y = 50$ on the chalkboard, then asking students to explain the meaning of the variables x and y in this equation and the relationship of the equation to the original problem, the teacher can enrich students' understanding of the concept of variables.

Next, by incorporating a coordinate graph to represent their numerical solutions, students can see that all their numerical solutions lie on the same line; that $5x + 20y = 50$ describes all points that lie on this line; and that a point on this line, namely, $(-2, 3)$, gives a surprising but quite reasonable method of making change for a fifty-dollar bill. See **figure 1.**

Revisit a setting familiar to students

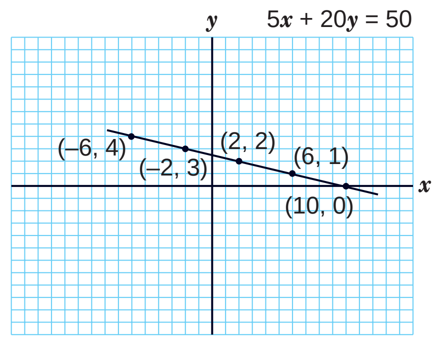

Fig. 1
A graphical representation of the "making change" problem

Students should be invited to describe in words, in writing or orally in class, a scenario for changing a fifty-dollar bill that matches the coordinate pair $(-2, 3)$. Notice how a problem that started out having a simple numerical-only solution has been enriched by the application of multiple representation.

Revisit rich or popular settings

Using or revisiting a familiar setting is a powerful algebra-for-all teaching strategy. In general, a new setting of a word problem can intimidate students. Students may not understand the meanings of some words, or situations may not be related to their everyday experiences or their culture. When new problem settings are presented to students, be alert for any signs that indicate that barriers are preventing students from entering or engaging mathematically with the problem. After class time has been invested to familiarize students with a problem setting, use the setting again, especially if it can be modified to enrich and extend their understanding of mathematics.

As an illustration, consider the following setting. A turtle walks at five feet per minute, and a snail crawls at three feet per minute. The turtle and the snail start from an oak tree and head toward an elm tree that is located thirty feet from the oak tree. In this simple setting, students can investigate, through multiple representations, patterns in words, tables, symbols, and graphs. By having students link these representations with one another, they experience a rich lesson in many of the big ideas of algebra.

When the class is familiar with this setting, use it again but with an additional twist: give the snail a nine-foot head start on the turtle. Students can then investigate how the various multiple representations have changed. This added condition raises the question of which critter reaches the elm tree first. The question can be solved using tables, setting up and solving an algebraic equation, or locating an intersection point of graphs. This multiple-representation problem-solving approach extends and deepens students' understanding of important concepts of algebra. In addition, the class time needed to cover these higher-level topics is reduced, since students are already familiar with the setting.

An additional variation on this setting is to start the turtle walking from the oak and the snail crawling from the elm toward each other. Again, students can investigate how the various multiple representations have changed as they attempt to solve where and when the critters meet. This twist introduces students to decreasing distance relative to the oak tree, negative rate of change, setting up and solving an algebraic equation, and intersecting graphs as a solution to an equation.

Finally, be alert for situations that are popular with students; such situations are often related to the latest school fads. Embedding a mathematical activity within a contextual setting that is linked to a school craze, fad, or even a school event is an effective way to engage students' interest in mathematics and its role in their lives.

Do not begin the year by remediating

With each new class, every teacher faces students whose mathematical knowledge is widely divergent. How can a teacher carry out her or his responsibility to effectively teach all students so that all reach high levels of learning by the end of the year? This perennial question has no simple answer.

However, planning a remediation effort for the first two to three weeks of the school year is fated to fail for several reasons. How can you decide what to review before you really know the deficiencies that your students have? Some may need a review of basic skills; but others may be deficient in such other prerequisites as graphing skills, proportional reasoning, the concept of a fraction, the concept of a variable, and number sense. Also, spending two or three weeks on review and remediation wastes valuable class time, and at the end of that time, students who had deficiencies in prerequisite knowledge continue to have them.

Instead of starting the school year with remediation activities, begin immediately with the curriculum that defines the course. Indeed, some students in your classroom will have deficiencies. But use the previously described "friendly" numbers that have been chosen to be within reach of their skill levels to involve them in activities that emphasize the big ideas of algebra and algebraic thinking. Then, as you introduce students to aspects of the curriculum, fold into your lessons any review of prerequisites that individual students may need. Thus, the strategy is to tailor remediation to the needs of your students without sacrificing your curriculum objectives.

Another tip for helping students in need of remediation is to use the first five to ten minutes of each class—a time when not much can be accomplished because of such activities as students changing classes, roll taking, and other classroom-management duties—to place a warm-up problem or activity on the chalkboard. Choose these activities carefully for their ability to incorporate needed review work in an interesting and conceptual way. As an example, consider the following as a warm-up problem.

> The value of the area of the square in **figure 2** is 1. What are the values of the areas of each of the pieces of the square?

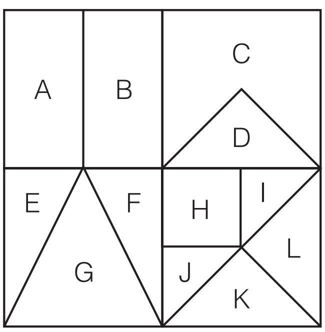

Fig. 2
A square with area 1

Although problems similar to the one above have been in the literature for a long time, they exemplify problems that are very conceptual and rich in content. The problem deals with proportionality, that is, the concept of a fraction, and with such elements of geometry as shapes and area. It also permits a variety of solution approaches that can lead to rich class discussions. Finally, this warm-up activity has tremendous potential for follow-up activities. For example, suppose that the area of the triangle labeled D is 1. What is the value of the area of each of the other pieces? As a matter of fact, any area value can be assigned to any piece of this square to construct a new problem that is based on this figure and the concept of fraction.

Involve students in guided explorations; use learning-by-discovery teaching strategies

To capture the essence of algebra, I like to define algebra as follows:

> Algebra is the *process* of organizing the *arithmetic* needed to find an answer to a question involving quantities that are not yet known.

According to this definition, algebra is a natural extension of arithmetic. Thinking of algebra as a curriculum strand for grades K–12 makes sense. Algebra is not just a list of topics in a textbook. Algebra is a process that is called *algebraic thinking*. Notice also that everyone needs algebra, because as people grow older, their quantitative questions become more complex by involving quantities that are not yet known, whether or not they have taken algebra in school. The reader is invited to compare this definition with three others that were showcased in the July/August 1997 issue of the *NCTM News Bulletin*.

Guided explorations or learning-by-discovery teaching strategies actively involve students in this process of organizing their arithmetic to find answers to questions. For example, when the previously described turtle-and-snail setting is presented as a guided-exploration lesson, students are involved in algebraic thinking and investigating a rich panorama of big ideas of algebra: arithmetic-sequence patterns; such multiple representations as data in tables, in graphs, in symbols, and in words; rate of change; slope; y-intercept; linear functions; evaluating functions; setting up and solving linear equations; and solving simultaneous equations in such multirepresentational ways as tables, graphs, and symbols. These approaches are more likely to keep students engaged and hence, to encourage them to construct a personal understanding of the mathematics.

Be a good listener

Learn to recognize correct thinking in students even when it may be incomplete or lacking in closure

In other words, be a good listener and be as alert to your students as you are to what you are teaching. A vignette about a mathematics teacher, whom we will call Mr. Relentless, helps illustrate this algebra-for-all strategy. This vignette is based on a real classroom event.

> Mr. Relentless gave his class the following problem on the big idea of proportionality: "Frank's car travels twenty-five miles in thirty minutes. What is the car's average speed?"

> The answer that Mr. Relentless expected to this question was fifty miles per hour.
>
> In class, he called on Kisha to give her answer to his question. Kisha responded, "The average speed of Frank's car is 5/6 mile per minute." Mr. Relentless brushed Kisha's answer aside as he asked whether anyone else in class could give the correct answer to this problem.
>
> Did Kisha understand proportionality and rate of change?

Be a teacher who listens carefully and who values diversity of thinking. Be a teacher who looks for understanding of the big ideas rather than just an answer that conforms exactly to an answer key.

Mold lessons, whenever possible, around the interests of individual students

The following situation actually occurred in a ninth-grade algebra class in a high school in an eastern urban inner city. Students in this ninth-grade class were presented with the following performance-based guided-exploration task.

> A farmer had 24 yards of fencing. What are the dimensions of a rectangular pen that gives his sheep maximum grazing area?

For this task, the students were equipped with graphing calculators and were assigned to work in groups of three.

As an entry activity, the teacher had students use coordinate grid paper to draw sample rectangular pens that met the conditions in the problem statement. The students were enthusiastically involved in this problem and were busily drawing sample rectangular pens on grid paper and noticing that the pens could be constructed in many ways. All students were engaged, that is, except for one young lady who had quietly detached herself from her group. She preferred, instead, to focus on her grooming. She fluffed her hair and checked her eye shadow and her nail polish. When asked why she was not interested in the problem, she commented respectfully, yet honestly, that she could not care less about sheep in a pen. As an inner-city youth, she was stating that the problem setting had no relationship to her world.

Algebra-for-all teachers teach students, not pages

Could this problem be changed so that it would be relevant to this young lady? Faced with this question, the teacher discovered that the student liked flowers, in particular, red roses. Homing in on this volunteered interest, the teacher restated the problem for Maria as follows:

> Maria has 24 yards of fencing, which she can use to make a rectangular garden for growing red roses. Each rosebush needs space to grow. A one-yard-by-one-yard square gives a rosebush sufficient growing room. What dimensions will allow her to plant the maximum number of rosebushes in this rectangular garden?

With this new setting, tailored to Maria's interest in roses, Maria smiled delightedly at the teacher and became involved in the activity.

Establish a safe classroom environment

A safe classroom environment is one in which a teacher establishes that—

- everyone in class, including the teacher, is part of a community of learners;
- taking risks on the part of students, by asking questions, volunteering answers, and so on, is encouraged and respected;
- different solution methods and different ways of thinking about problems and mathematics are valued and give the teacher instructional opportunities; and
- collaboration with classmates is valued.

CONCLUSION

In closing, consider for a moment the following problem.

> Mr. Relentless, a mathematics teacher, is planning his algebra course for the school year. He has 180 days to cover the new textbook. The book is 983 pages long. How many pages, on the average, must he cover each class day to complete the course by the end of the year?

An answer to this problem that sums up what it means to be an algebra-for-all teacher is that algebra-for-all teachers teach students, not pages.

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