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PRESENTS

Is It Fair?

A Look at Probability

by Marilyn Burns

For the Teacher

This poster suggests an investigation that engages students with ideas about probability and statistics. Many important decisions are made in the face of uncertainty and in areas where variability is expected—weather predictions, economic forecasts, budget decisions, or simply planning when to leave to arrive at a destination on time. Many of these decisions involve mathematical ideas that require thinking about probability and statistics. Students benefit from learning about these areas of mathematics at all levels, beginning in the elementary grades.

"Match or No Match" can be investigated at all grades. Although all students can make predictions and try out their ideas for each version, thinking in depth about the probabilities will not be appropriate for young children. The activity can stimulate their mathematical thinking, however, and give them informal experience with important mathematical ideas.

The Mathematics of Fair Games

It's important to think about the idea of fairness over time. That is, if a game has a fixed number of turns or a time limit for playing, there is always the chance, even when a game isn't fair, that either player will win. The idea is that if many games are played, or if one game is played for a length of time, a pattern emerges that will make it clear whether or not the game is fair. In a fair game, whichever player might win or be ahead at any given moment will teeter back and forth among players, and it will seem that any player can win. In an unfair game, players might win or be ahead at any moment, but one player has the advantage and, over time, will prevail. Mathematically, it's the "over time" notion that is important—the idea that over many trials, it will become evident whether a game is fair or unfair. Since it's not possible to play a game for an infinite length of time, mathematical theories play an important role. This investigation promotes students' thinking and reasoning mathematically.

Note: The ideas on this poster are adapted from *Math By All Means: Probability, Grades 3-4*, a replacement unit by Marilyn Burns that provides lessons and activities for six weeks of instruction. The "Match or No Match" game presented on this poster is one of the activities from the unit.

In the Classroom

Introducing the Idea of Fair Games

Have a class discussion in which students tell what they do when they need to decide among themselves who chooses an activity or goes first in a game. Relating an activity to students' own experiences not only helps to motivate their interest but also gives teachers a way to build on students' prior knowledge.

After all students have had the chance to give their ideas, tell them about mathematicians' notions of fairness in games. You may want to read the information from the front of the poster to the class.

As an introduction to the "Match or No Match" activity, play the games of "Heads and Tails" and "Red and Blue" with the class. The important idea is that although one player or the other may win in either game, both players have a fair shot at winning in "Heads and Tails." In "Red and Blue," however, the game is skewed when there are more tiles of one color than the other.

Presenting "Match or No Match"

Use version 1 of "Match or No Match" to introduce the game. Put two blue tiles and one red tile in a paper bag. Focus students' attention on investigating whether or not the game is fair, not on which player wins. To do this, do not let students choose to be match or no match players. Instead, have them work together to test each version by putting the correct number and color of tiles in a bag, and then reach in to remove two of them and note whether or not there is a match.

Ask a student helper to come to the front of the room. Draw two tiles from the bag and show them to the class. Have your student helper make a tally mark to indicate whether the draw is a match or not. Then replace the tiles. Continue drawing (or have other students in the class draw) while the student helper continues to make tally marks. Do this for 20 draws.

Match	𝍸
No Match	𝍸 //

Tell the students that, working in pairs, they are to test the game this way, taking at least 20 samples for each version.

After students have completed collecting data, have them report the number of match and no match draws for each version. Collect this information and have students compute the totals.

Ask students to discuss what the data shows and how students might explain the results. If you wish, students may write about the experience, explaining why or why not each version is fair.

About the Mathematics of "Match or No Match"

Of the three versions of the game, only one is mathematically fair—Version 3. This seems counterintuitive to many students (and adults), and the data from the class may or may not support it. It's important to remember that collecting data from the students into a larger sample size provide more reliable information than do individual results. Although the empirical data provide useful information for thinking about and discussing the game, the data are not sufficient to prove a theory.

There are different ways to think about the games. One is to list the possibilities of drawing two tiles from the bag. In Version 1, for example, with two blue tiles and one red, it's helpful to describe them as

B1, B2, and R1. With this, three combinations of two tiles are possible:

B1, B2	**M**
B1, R1	**NM**
B2, R1	**NM**

Of these three, one is a match (M) and two are no matches (NM). Mathematically, there is a 2/3 chance of getting a no match. Therefore, the game is not fair.

Another way to analyze the game is with a tree diagram. In this case, you imagine drawing first one tile and then another. (As long as you don't return the first tile before drawing the second, this is the same as drawing two tiles at the same moment.) If you draw B1 first, two possibilities exist for the second draw—B2 or R1. The same holds true for any tile drawn first; two possibilities always exist for the next draw. This can be represented as follows:

B1 — B2 **M**
B1 — R1 **NM**

B2 — B1 **M**
B2 — R1 **NM**

R1 — B1 **NM**
R1 — B2 **NM**

This results in a 4/6 chance of a no match, which is the same as the 2/3 chance shown the other way.

You'll have to decide how much of this information to give to students, depending on their age, interest, and mathematical ability. Hold off from any formal presentation, however, until students have had ample time to investigate, think, discuss, and write about their ideas.

Inventing New Games— An Extension

One way to continue students' study of the mathematics of fair games is to have them invent their own games. Students can then analyze one another's games and decide if they're fair or unfair.

Have students describe their invented games as follows:

1. *Name and type of game.* The game must be a game of chance for two or more players. Students should give the game a name that is descriptive and original.

2. *Number of players.* Students should specify the exact number of players or the minimum and maximum numbers of players.

3. *Materials needed.* They should choose common materials or include directions for materials that need to be made.

4. *Rules for play.*

5. *Explanation of fair or unfair.* They should indicate whether or not the game is fair and explain their thinking.

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Invent a Game Worksheet

Name of game

Authors

Number of players

Materials needed

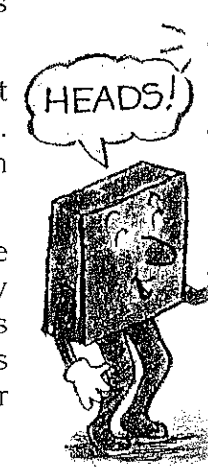
Rules for play

This game is ☐ fair ☐ unfair because

A Look at Probability

What do you think makes a game a "fair" game? To mathematicians, a fair game is a game of chance that gives all players the same opportunity to win. That's because the game depends totally on luck. It doesn't matter who goes first, or who is older, taller, stronger, faster, or more skilled. It's all chance.

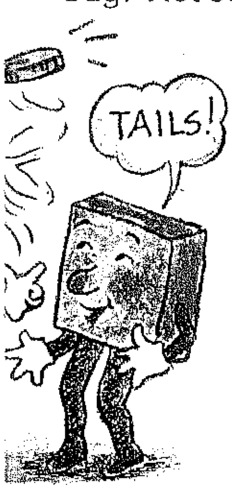
Here's an example of a mathematician's idea of fair. When two people want to decide which one gets the first pick of cookies, they sometimes flip a coin. Because heads and tails have the same chance of coming up, flipping a coin is a fair method. The lucky person gets first pick.



A fair game works the same. Suppose two people decide to play a game called "Heads and Tails." One chooses heads, the other chooses tails, and they take turns tossing a coin. Each time the coin comes up heads, the heads player scores a point. Each time the coin comes up tails, the tails player scores a point. Although one player or the other will win, neither one has a better chance. "Heads and Tails" is a fair game.

Just being a game of chance, however, isn't enough to make the game fair. Suppose, for instance, that it's raining, with no electricity and nothing good to eat, and you and your friend have had enough of "Heads and Tails." So you invent a new game—"Red and Blue." One player is "red"; the other is "blue." To play, you put red and blue tiles in a paper bag and take turns reaching in (without looking), removing a tile, and then replacing

it. Each time a red tile is removed, the red player scores a point; if the tile is blue, the blue player scores a point. Sound fair? Well, what if two red tiles and one blue tile were in the bag? Not so fair. The player who picks red would have the advantage, for sure. Mathematicians would say that the red player has twice the chance of winning, or that the probability of red winning is two out of three; that is, P(red) = 2/3. (Can you explain why this makes sense?)



Fair or Not Fair?

Another game like "Red and Blue" is called "Match or No Match." In this game, one person is the "match" player and the other is the "no match" player. Again, red and blue tiles are placed in a bag. When players reach in, however, each removes two tiles. If the colors are the same (two reds or two blues), the match player scores; if the colors are different (one of each color), the no match player scores. Is this game fair? Or does one of the players have a better chance? The correct answer to these questions is: IT DEPENDS ON HOW MANY OF EACH COLOR ARE IN THE BAG!

So, here are three possible versions of what might be in the bag:

Version 1 ■ ■ ■ Version 2 ■ ■ ■ ■ Version 3 ■ ■ ■ ■

Your job is to figure out if any of these versions are fair games. Your teacher will tell you how to get started with the investigation.