CUBE SCULPTURES

- Surface area
- Volume
- Spatial visualization

Getting Ready

What You'll Need

Snap Cubes, about 70 per pair

Isometric dot paper, page 118

Activity Master, page 106

Overview

Students search to find all possible surface areas that can be created by building structures made from 16 Snap Cubes. They then investigate the dimensions and volumes of rectangular prisms that could be used to enclose their structures. In this activity, students have the opportunity to:

- find the surface area of a variety of structures
- discover that structures with the same volume may have different surface areas
- discover that different-looking structures that have the same volume may have the same surface area
- recognize that elongated structures have a greater surface area than compact structures that have the same volume
- determine the overall dimensions of an irregular-shaped structure

Other *Super Source* activities that explore these and related concepts are:

Storage Boxes, page 49

Wrapping Paper, page 57

The Activity

On Their Own (Part 1)

The students in the eighth grade art class are using recyclable materials to build cubes which they will join together to make sculptures for the school courtyard. Each sculpture will be made from 16 cubes. The visible faces of each cube will be painted in different colors. How many different colors are needed?

- Work with a partner. Use Snap Cubes to design models of several different sculptures, each containing 16 cubes.
- Determine the number of colors that would be needed to paint each sculpture. Remember, each visible face must be a different color.
- Record your sculptures on isometric dot paper. Record the volume and surface area of each sculpture using the edge of one cube as the unit of measure.

- Now try to make models of sculptures that will require different numbers of colors from those you recorded. When you find one, record it as you did before.
- Continue until you think you've modeled at least one sculpture for every possible number of visible faces. Be ready to discuss your findings.

Thinking and Sharing

Ask students to reconstruct the sculptures they built requiring the smallest number of different paint colors. Do the same for their sculptures requiring the greatest number of different paint colors. Then have students help you list the surface areas (from smallest to greatest) of the different sculptures they built.

Use prompts like these to promote class discussion:

- What did you notice as you built models of different sculptures?
- How do the volumes of your sculptures compare?
- How do the surface areas of your sculptures compare?
- How did you go about building models of sculptures that would require different numbers of colors?
- Did you notice any patterns as you worked? Were the patterns helpful in building new sculptures? If so, explain.

On Their Own (Part 2)

What if... *the students decide that each sculpture is to be enclosed in a clear plastic rectangular prism that will protect it from the weather. What size prisms will they need to construct?*

- Reconstruct one of your sculpture models. Imagine enclosing it in the smallest possible rectangular prism that could hold it.
- Determine the dimensions and volume of the prism. Record these measurements near the drawing of your sculpture.
- Reconstruct each of your other models and determine the dimensions and volumes of the prisms they would require. Record your findings.
- Compare your different models and the dimensions of their enclosing prisms. Be ready to discuss your observations.

Thinking and Sharing

Ask students to recreate several sculptures that required rectangular prisms with different dimensions. Have students display their sculptures, discuss how they determined the size of the enclosing prisms, and record the dimensions and corresponding volumes on the chalkboard.

Use prompts like these to promote class discussion:

- How did you go about determining the size of the rectangular prisms?
- Were some prisms easier to visualize than others? If so, which ones, and why?
- Were some of the prisms harder to visualize than others? If so, which ones and why?
- What was the size of the smallest prism you needed? How did this prism compare in volume to the sculpture it would enclose?
- What was the size of the greatest prism you needed? How did this prism compare in volume to the sculpture it would enclose?
- What other observations did you make about your models and their prisms?

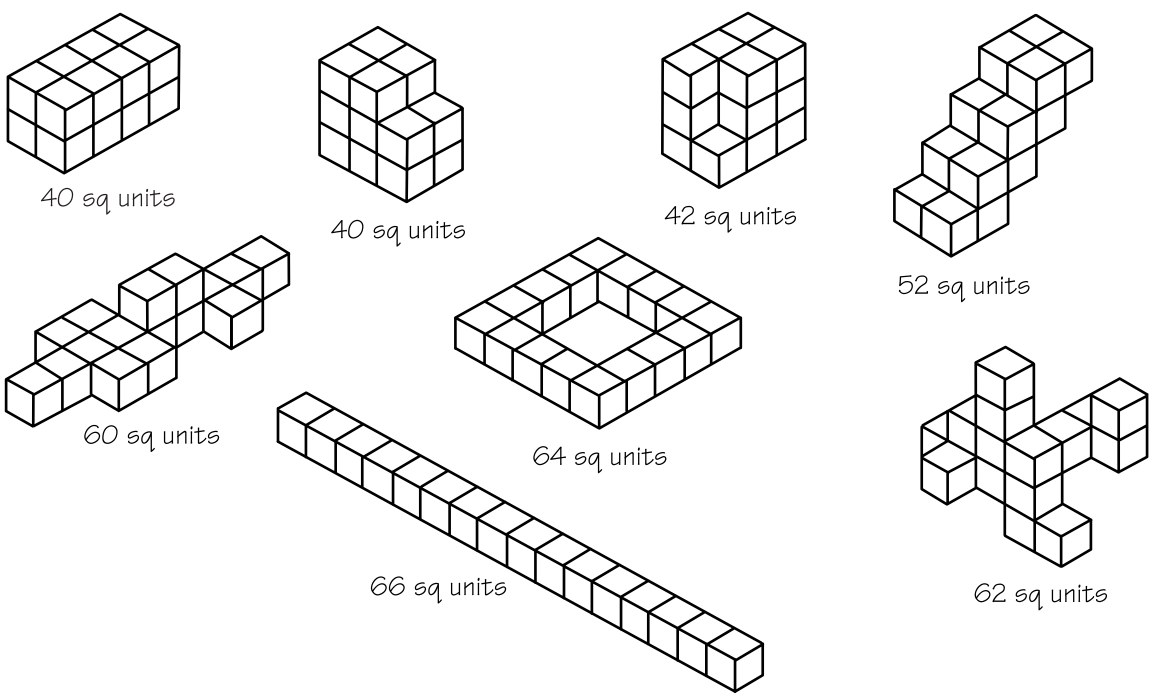
Suppose you were building sculptures made from 20 cubes. Describe how you would construct the models having the smallest and greatest possible surface areas. Explain how you know that no other sculpture could have a smaller (or greater) surface area than the ones you described.

Teacher Talk

Where's the Mathematics?

It is often the case that students believe that shapes with the same volume have the same surface area. Many students also believe that shapes that look totally different from each other must have different surface areas. As they investigate the problem set forth in Part 1 of this activity, students will be able to discover that both of these ideas are erroneous.

There are 14 different possible surface areas of structures that can be created using 16 Snap Cubes. They range in area from 40 square units to 66 square units. Some examples of structures that students might build and record are shown below with their respective surface areas.



Students may notice that although all of their structures have the same volume (16 cubic units), the structures that are more compact have smaller surface areas than those that are elongated. They may recognize that this is due to the fact that in compact shapes, more cube faces are "hidden" inside the structure and, therefore, do not contribute to the surface area. To build new structures requiring different numbers of paint colors, students may take some of their more compact models, and, by rearranging some of the cubes, "uncover" some of the hidden faces, thereby increasing the surface area.

Students may also notice that the number of different colors needed to paint each of their structures is always an even number. In fact, for every even number from 40 to 66, it is possible to build at least one structure requiring that number of different paint colors. Students may want to investigate why the structures always require an even number of paint colors. As they examine their structures, they may discover that the number of cube faces that are exposed to the front of the structure is equal to the number exposed to the back; the number of cube faces that are exposed to the left of the structure is equal to the number exposed to right; and the number of cube faces that are exposed to the top of the structure is equal to the number exposed to the bottom. Since these numbers occur in pairs, their sum (the total surface area) will always be an even number.

In Part 2, students need to use spatial visualization to picture the smallest rectangular prism that will contain each of their structures. In doing this, they are finding the overall dimensions of their models. Students should recognize that their prisms will need to accommodate the longest row of cubes from each of the three dimensions of their structures. The volume of each prism can then be calculated by multiplying these three measurements together.

One particularly interesting result is that the prisms needed to enclose the structure with smallest surface area (the 2 x 2 x 4 prism) and the structure with greatest surface area (the 1 x 1 x 16 prism) have the same volume (16 cubic units). These are the smallest prisms that will hold a 16-cube structure. Students may notice that in these cases, there is no empty space inside the enclosure once the structure is placed inside. Therefore, the volumes of these enclosures are equal to the volumes of the structures themselves. Larger enclosing prisms will be needed for the other structures, all of which will contain empty space once the structure is placed inside. The structure below would require the largest enclosure, a 6 x 6 x 6 prism, having a volume of 216 cubic units, 200 of which will be empty space once the structure is placed inside.