Name Date

Student Sheet 8

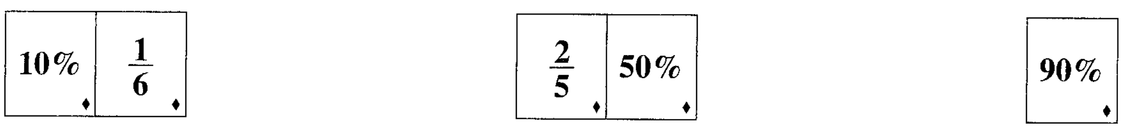
How to Play the In-Between Game

Materials: Fraction Cards—diamond (♦) cards only, Completed Percent Equivalents Strip (for reference only)

Players: 2

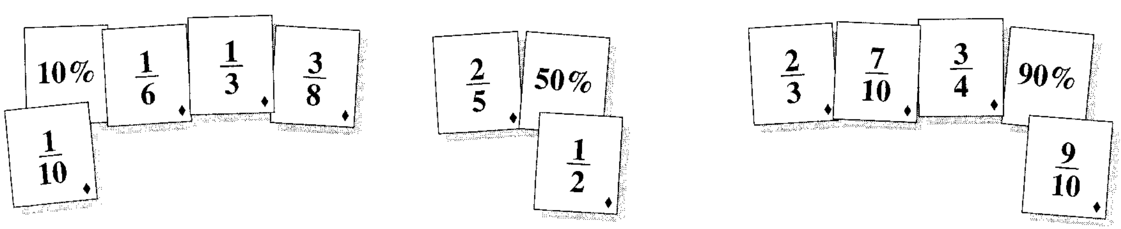
1. Place the 10%, 50%, and 90% cards on the table (see picture).
2. Mix the Fraction Cards. Deal six to each player.
3. Players take turns placing a card so it touches another card. You may place a card to the right of 10%, on either side of 50%, to the left of 90%, or on top of any percent. As you play a card, state the fraction and its percent equivalent.

   For example, if you place the $\frac{1}{6}$ card to the right of 10%, you would say, "One-sixth is $16\frac{2}{3}$%."
4. Cards must be placed in increasing order, from left to right.



   A card may *not* be placed between two cards that are touching.

   In this example, the $\frac{1}{8}$ card may **not** be placed between the $\frac{1}{6}$ and the 10% cards. So, you can't place it in this round.
5. Your goal is to place as many cards as you can. The round is over when neither player can place any more cards. Your score is the number of cards left in your hand.

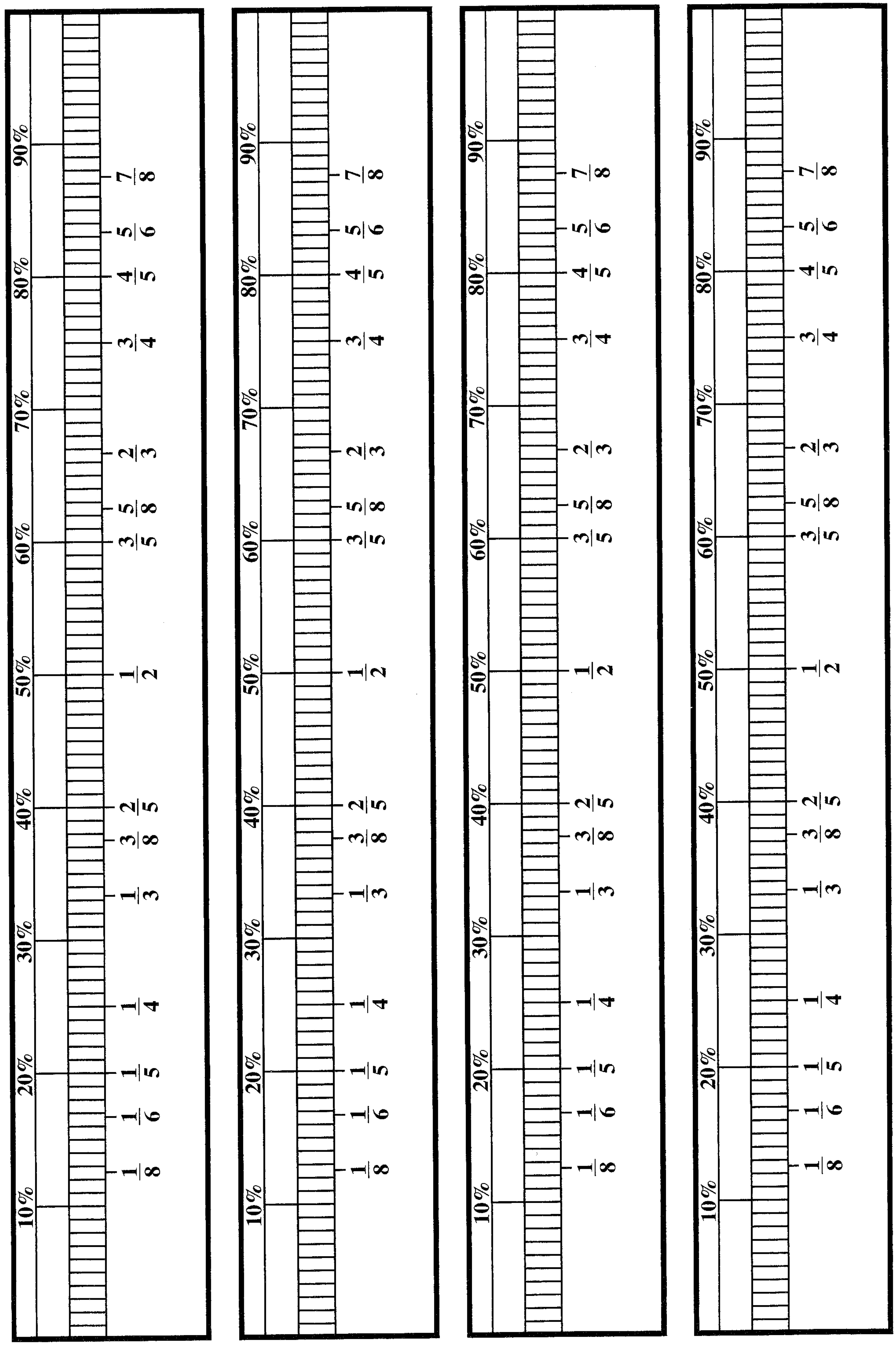


   At the end of a round, the table might look like this:

   Player 1 could not place $\frac{1}{8}$ and $\frac{4}{5}$ and so has a score of 2.
   Player 2 used all six cards and so has a score of 0.
6. At the end of five rounds, the player with the lowest score wins.

COMPLETED PERCENT EQUIVALENTS STRIPS

Cut out four strips along heavy black lines. Each group uses one to check their own strips.



$\frac{1}{2}$ ♦	$\frac{1}{3}$ ♦	$\frac{2}{3}$ ♦	$\frac{1}{4}$ ♦
$\frac{3}{4}$ ♦	$\frac{1}{5}$ ♦	$\frac{2}{5}$ ♦	$\frac{3}{5}$ ♦
$\frac{4}{5}$ ♦	$\frac{1}{6}$ ♦	$\frac{5}{6}$ ♦	$\frac{1}{8}$ ♦
$\frac{3}{8}$ ♦	$\frac{5}{8}$ ♦	$\frac{7}{8}$ ♦	$\frac{1}{10}$ ♦
$\frac{3}{10}$ ♦	$\frac{7}{10}$ ♦	$\frac{9}{10}$ ♦	50% ♦

10% ♦	90% ♦	$\frac{2}{2}$	$\frac{3}{2}$
$\frac{3}{3}$	$\frac{4}{3}$	$\frac{2}{4}$	$\frac{4}{4}$
$\frac{5}{4}$	$\frac{6}{4}$	$\frac{5}{5}$	$\frac{6}{5}$
$\frac{7}{5}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$
$\frac{6}{6}$	$\frac{7}{6}$	$\frac{8}{6}$	$\frac{9}{6}$

If students are making major errors, suggest that they discuss with neighbors some of the approaches they are using to mark the strips and why these approaches may or may not make sense.

When students have completed their Percent Equivalents Strips, distribute one Completed Percent Equivalents Strip to each group of four students. Students share these as they check their own strips and make them more precise. The students will be using these strips as reference tools throughout the unit, so have a few blank strips available for students who have made many errors and are determined to start over (but do not encourage this).

Activity

Playing the In-Between Game

Hand out a prepared deck of Fraction Cards to each pair of students. They take out the set of cards with diamonds (♦) to play the In-Between Game. Students also will need their completed Percent Equivalents Strips for reference during the game.

You might introduce this game by leading two students through a demonstration game, with the rest of the class grouped around to observe. Seat the two students side by side so both can easily see the cards on the table. In playing later, they will keep their dealt hands concealed from one another, but that is not necessary for the demonstration game since the observers will be able to see both hands.

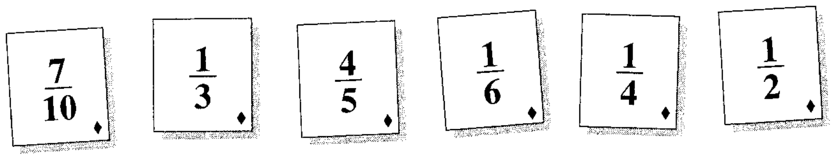
The rules are given on Student Sheet 8, How to Play the In-Between Game. Refer to this sheet as needed while teaching the game. Decide whether or not you want to distribute the rules now for student use in class; in either case you will send them home with students to help them teach the game to their families.

The basic rules are as follows. Start by placing the three percent cards (10%, 50%, and 90%) spread apart in a row across the table (10% on the left, 90% on the right). (There should be plenty of room between the percent cards to place several fraction cards.) Shuffle the Fraction Cards and deal 6 to each player.

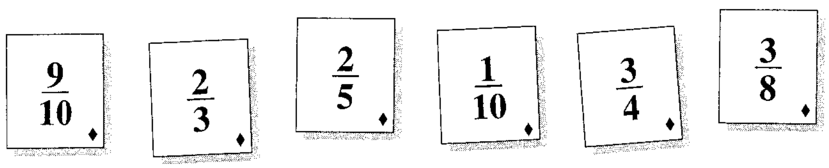
Players take turns placing one of their cards on the table. Each card placed must touch a card that is already on the table. Players may not place a card between two adjacent cards that are touching. Any card that is equivalent to one of the percent cards may be placed on top of the appropriate percent. At all times during the game, all the cards on the table must be in ascending order, from left to right. Play continues until neither player can place another card. The number of cards left in a player's hand is that player's score for the round. After five rounds, the player with the lowest score wins.

A sample round will illustrate how the game is played.

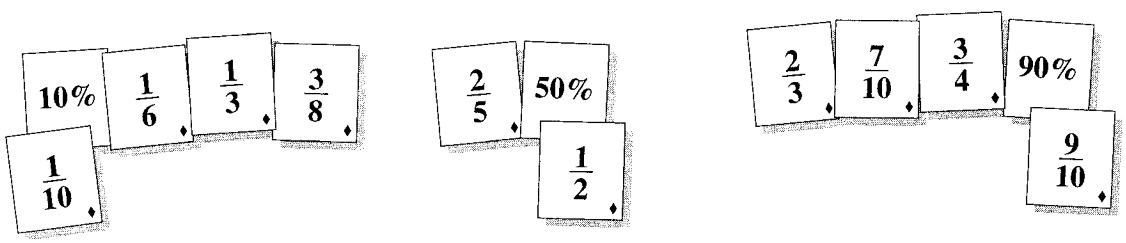
Jasmine is dealt these six cards:



Maricel is dealt these six:



At the end of the round, the table looks like this:



Jasmine has been unable to place 1/4 and 4/5. She ends the round with a score of 2, while Maricel finishes with a score of 0.

Observing the Students When students understand the rules, they play the In-Between Game in pairs. Circulate and observe students as they play.

- How are they figuring out equivalent fractions and percents? Are they using their Percent Equivalents Strip? Do they automatically know the percent equivalents for some fractions?
- Are students monitoring and checking each other's play?
- Are students developing strategies as they play the game?

Variations

- Students might enjoy playing a collaborative version with their cards exposed, working together so that both players place all their cards.
- Challenge students to play without referring to their Percent Strips.
- Pairs stop after a game and ask another team to check their work.
- Students might post their game on a transparency. You can then use a few examples to analyze some sample rounds with students.

Name Date

Student Sheet 21

Fraction to Decimal Division Table

Numerator

Denominator

N/D	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

When students have had a few minutes to compare their tables, call them together to share with the whole class some of the ways they determined entries without a calculator, and why some of the patterns they noticed occur. One way you might do this is to ask students what patterns they found useful and make a list. Then ask them to work in pairs or small groups to write why these patterns occur. Write out some of the patterns in your words, ending with "because…" for the students to complete. For example:

- The number 1 is down the diagonal because… *the number is divided by the same number and that's always 1.*
- As you read across any row, the numbers get larger because… *the top number is bigger so there are more of the decimal pieces. It's like when we did the percent lists. 20 percent is 1/5, and 40 percent is 2/5.*
- As you read down any column, the numbers get smaller because… *the denominator is bigger. You make more pieces so the pieces are smaller.*
- Sixths and twelfths are the only fractions in the table with both "long" and "short" decimals because… *when you divide by a number that multiplies to 12, like 4, it stops, but numbers like 7 always make long decimals.*

Name

Date

Student Sheet 21

Fraction to Decimal Division Table

Numerator

Denominator

N/D	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
3	0.3333	0.666	1	1.333	1.666	2	2.333	2.666	3	3.333	3.666	4
4	0.25	0.5	0.75	1	1.25	1.50	1.75	2	2.25	2.50	2.75	3
5	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
6	0.166	0.333	0.5	0.666	0.8333	1	1.1666	1.333	1.5	1.666	1.8333	2
7	0.1428571	0.2857142	0.4285714	0.5714285	0.7142857	0.8571428	1	1.1428571	1.2857142	1.4285714	1.5714285	1.7142857
8	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1	1.125	1.25	1.375	1.5
9	0.1111	0.222	0.333	0.444	0.555	0.666	0.777	0.888	1	1.11111	1.222	1.333
10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
11	0.0909	0.1818	0.2727	0.3636	0.4545	0.5454	0.6363	0.7272	0.8181	0.9090	1	1.0909
12	0.08333	0.16666	0.25	0.33333	0.416666	0.5	0.583333	0.6666	0.75	0.8333	0.91666	1

163

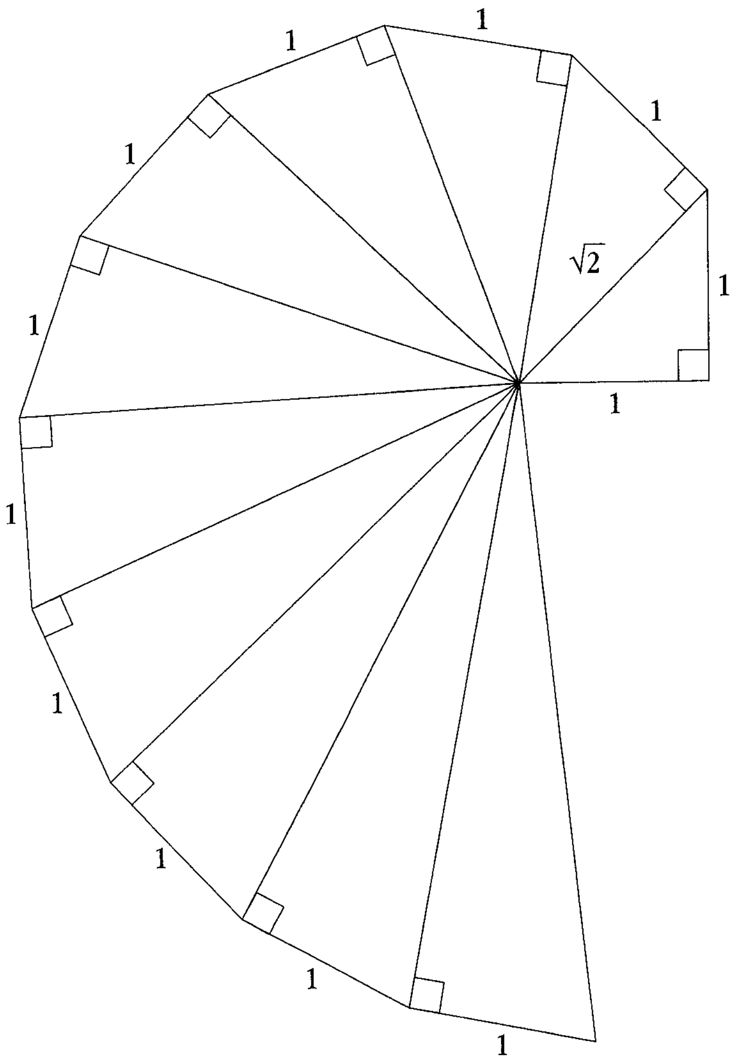
Investigation 3 • Sessions 5-6
Name That Portion

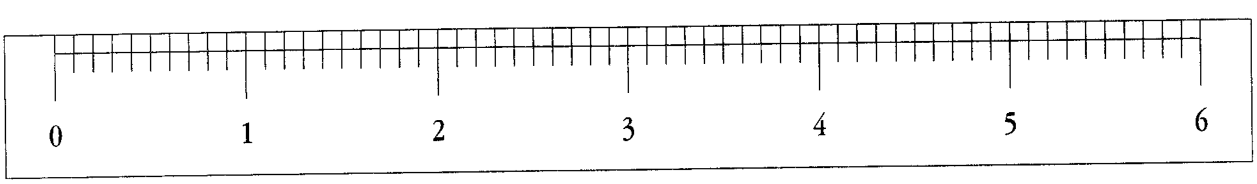
Chip Board

Chip Board

Labsheet 5.1

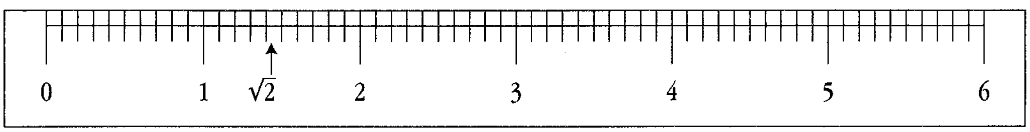
The Wheel of Theodorus





A. Use the Pythagorean Theorem to find the length of each hypotenuse in the Wheel of Theodorus. Label each hypotenuse with its length. Use the $\sqrt{}$ symbol to express lengths that are not whole numbers.

B. Cut out the ruler from Labsheet 5.1. Measure each hypotenuse on the Wheel of Theodorus, and label the point on the ruler that represents its length. For example, the first hypotenuse length would be marked like this:



C. For each hypotenuse length that is not a whole number, give the two consecutive whole numbers between which the length is located. For example, $\sqrt{2}$ is between 1 and 2.

D. Use your completed ruler to find a decimal number that is slightly less than each hypotenuse length and a decimal number that is slightly greater than each hypotenuse length. Try to be accurate to the tenths place.

5.1

Analyzing the Wheel of Theodorus

At a Glance

Grouping: ***small groups***

Launch

- Talk with the class about finding a decimal equivalent for $\sqrt{2}$.
- Introduce the Wheel of Theodorus.
- Have groups of two to four work on the problem and follow-up.

Explore

- Have each student label a number-line ruler.
- As students work, check on their understanding of measuring lengths and writing decimals.

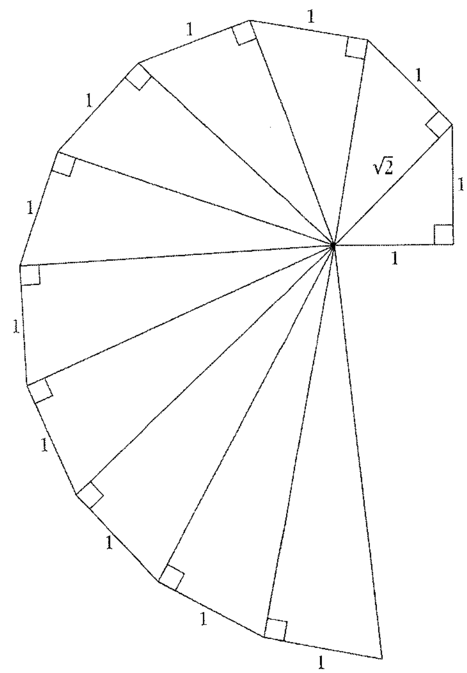
Summarize

- Display the Wheel of Theodorus, and ask students for the hypotenuse lengths.
- Talk about decimal equivalents for various square roots.

5.1 Analyzing the Wheel of Theodorus

In earlier investigations, you drew segments with lengths of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, and so on by drawing squares with whole-number areas. In this problem, you will investigate an interesting pattern of right triangles called the Wheel of Theodorus. The pattern will suggest another way to draw segments with lengths that are positive square roots of whole numbers.

The Wheel of Theodorus begins with a triangle with legs of length 1 and winds around counterclockwise. Each triangle is drawn using the hypotenuse of the previous triangle as one leg and a segment of length 1 as the other leg. To make the Wheel of Theodorus, you only need to know how to draw right angles and segments of length 1.



The Wheel of Theodorus is named for its creator, Theodorus of Cyrene. Theodorus was a Pythagorean and one of Plato's teachers.

Assignment Choices

ACE questions 9–20 and unassigned choices from earlier problems

Answers to Problem 5.1

A. The lengths of the hypotenuses from least to greatest are $\sqrt{2}$, $\sqrt{3}$, 2, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, 3, $\sqrt{10}$, $\sqrt{11}$, and $\sqrt{12}$.

B. See page 63g.

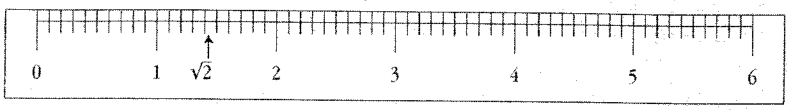
C. The lengths $\sqrt{2}$ and $\sqrt{3}$ are between 1 and 2; $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$ are between 2 and 3; and $\sqrt{10}$, $\sqrt{11}$, and $\sqrt{12}$ are between 3 and 4.

D. The length $\sqrt{2}$ is between 1.4 and 1.5; $\sqrt{3}$ is between 1.7 and 1.8; $\sqrt{5}$ is between 2.2 and 2.3; $\sqrt{6}$ is between 2.4 and 2.5; $\sqrt{7}$ is between 2.6 and 2.7; $\sqrt{8}$ is between 2.8 and 2.9; $\sqrt{10}$ is between 3.1 and 3.2; $\sqrt{11}$ is between 3.3 and 3.4; and $\sqrt{12}$ is between 3.4 and 3.5.

Problem 5.1

A. Use the Pythagorean Theorem to find the length of each hypotenuse in the Wheel of Theodorus on Labsheet 5.1. Label each hypotenuse with its length. Use the $\sqrt{\ }$ symbol to express lengths that are not whole numbers.

B. Cut out the ruler from Labsheet 5.1. Measure each hypotenuse on the Wheel of Theodorus, and label the point on the ruler that represents its length. For example, the first hypotenuse length would be marked like this:



C. For each hypotenuse length that is not a whole number, give the two consecutive whole numbers between which the length is located. For example, $\sqrt{2}$ is between 1 and 2.

D. For each hypotenuse length that is not a whole number, use your completed ruler to find a decimal number that is slightly less than the length and a decimal number that is slightly greater than the length. Try to be accurate to the tenths place.

Problem 5.1 Follow-Up

1. In Problem 5.1, you used a $\sqrt{\ }$ symbol to express the hypotenuse lengths that were not whole numbers. Use your calculator to find the value of each square root, and compare the result to the approximations you found in part D.

2. When Joey used his calculator to find $\sqrt{3}$, he got 1.732050808. Geeta says that Joey's answer must be wrong because when she multiplies 1.732050808 by 1.732050808, she gets 3.000000001. Why do these students disagree?

```
√3
      1.732050808
```

```
1.732050808*1.73
2050808
      3.000000001
```


Answers to Problem 5.1 Follow-Up

1. $\sqrt{2}$ = 1.414213562, $\sqrt{3}$ = 1.732050808, $\sqrt{4}$ = 2, $\sqrt{5}$ = 2.236067978, $\sqrt{6}$ = 2.449489743, $\sqrt{7}$ = 2.645751311, $\sqrt{8}$ = 2.828427125, $\sqrt{10}$ = 3.16227766, $\sqrt{11}$ = 3.31662479, $\sqrt{12}$ = 3.464101615; The numbers obtained using the ruler and the calculator are both approximations, but the calculator gives greater accuracy.

2. Both students have a valid point. Joey's number is an estimate accurate to eight decimal places, while Geeta is correct in pointing out that the square of this decimal approximation does not equal exactly 3. (Note: It would be impossible to write all the decimal places or to indicate any pattern in the decimal expansion for $\sqrt{2}$. This idea will be explored in the next two problems.)

Box Project

MATERIALS: String (some type of yarn, string, thread, etc.)
Index Cards
Magnet
Ruler
Scissors

1. You must construct the six boxes listed below. Each box must be a different color. (White is a color.)
2. All measurements are in inches. (Be very careful with the measuring.)
3. Cut out the required number of pieces and construct each box.
4. The last two boxes will be closed. (all six sides)
5. Tape, write, or put label of the name of each box on the side. (Same side on each box.)
6. Decorate appropriately. (You may be creative.)
7. Punch a tiny hole in the center of the bottom of each box.
8. Thread string, yarn, fishing line or some type of material through each hole.
9. Tie a knot in the string at the bottom of each box to keep the box in place.
10. Connect a magnet to the top end of the string. (These will hang from the ceiling with the magnet)
11. Write a definition sheet for Real numbers, Rational numbers, Integers, Whole numbers, Natural numbers, and Irrational numbers.
12. Write a paper that describes (in your own words) what you did to complete the project. Also tell how these all fit together.
13. Put your name and hour on the bottom of each box in ink.
14. Bring you project to school with the boxes stacked inside each other (string attached)

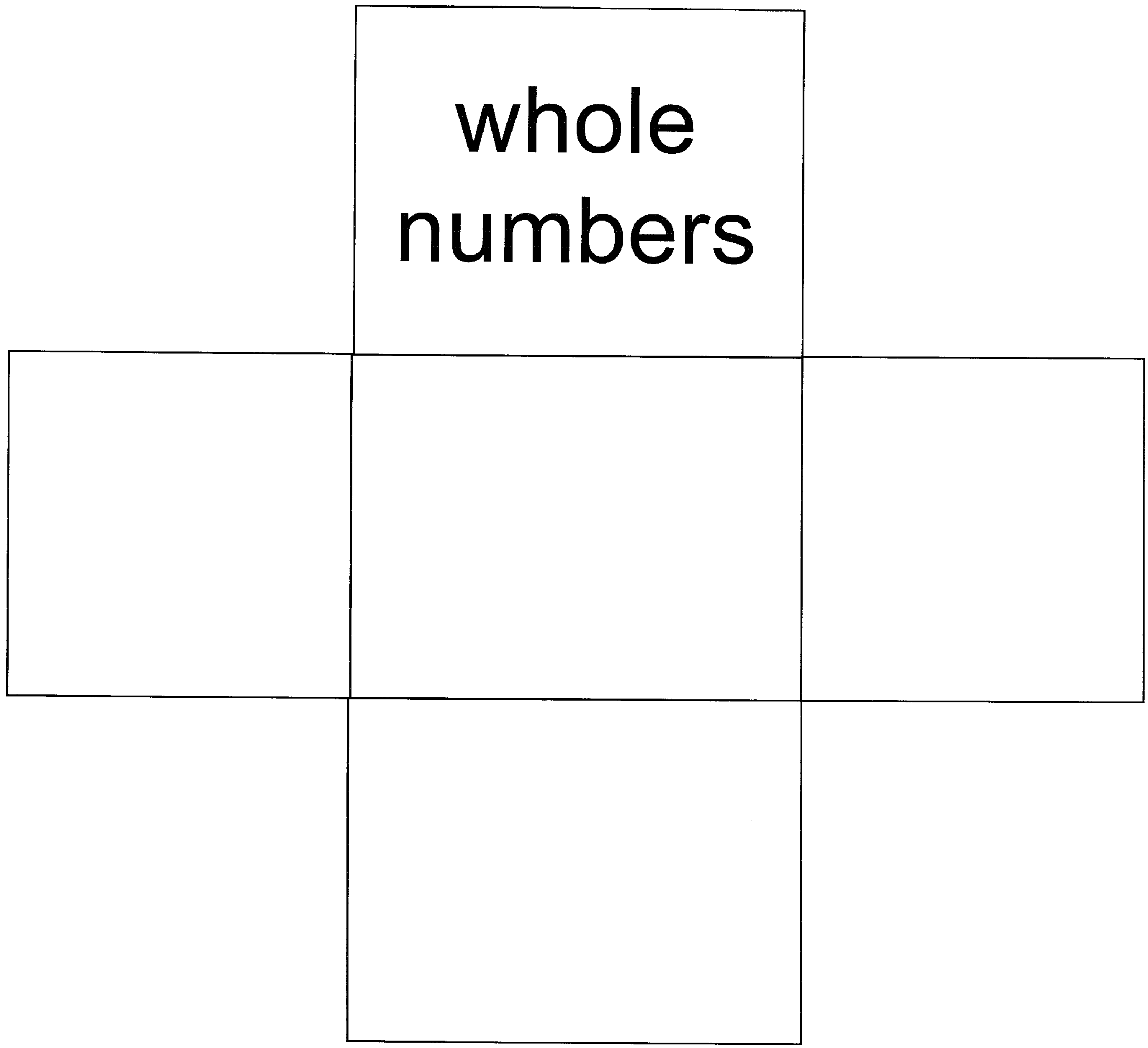
<u>Boxes must be made to these dimensions</u>:

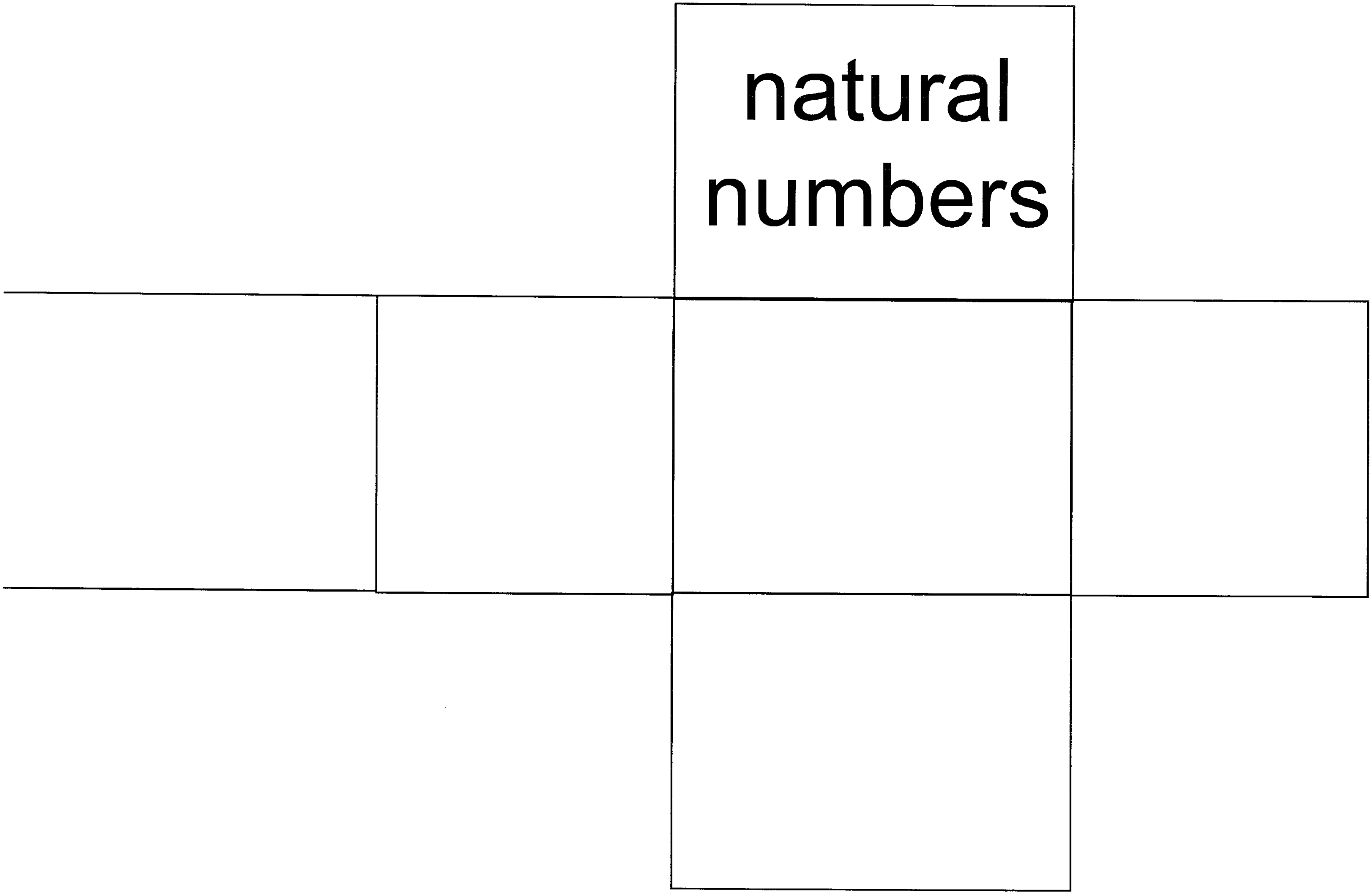
Real Numbers :	3 pieces	5 x 3
	2 pieces	3 x 3
Rational Numbers:	3 pieces	2 14/16 x 3 ¾
	2 pieces	2 14/16 x 2 14/16
Integers:	3 pieces	2 ¾ x 3 ½
	2 pieces	2 ¾ x 2 ¾
Whole Numbers	3 pieces	2 ½ x 3 ¼
	2 pieces	2 ½ x 2 ½
Natural Numbers	4 pieces	2 ¼ x 3
	2 pieces	2 ¼ x 2 ¼
Irrational Numbers	4 pieces	1 x 2 14/16
	2 pieces	2 14/16 x 2 14/16

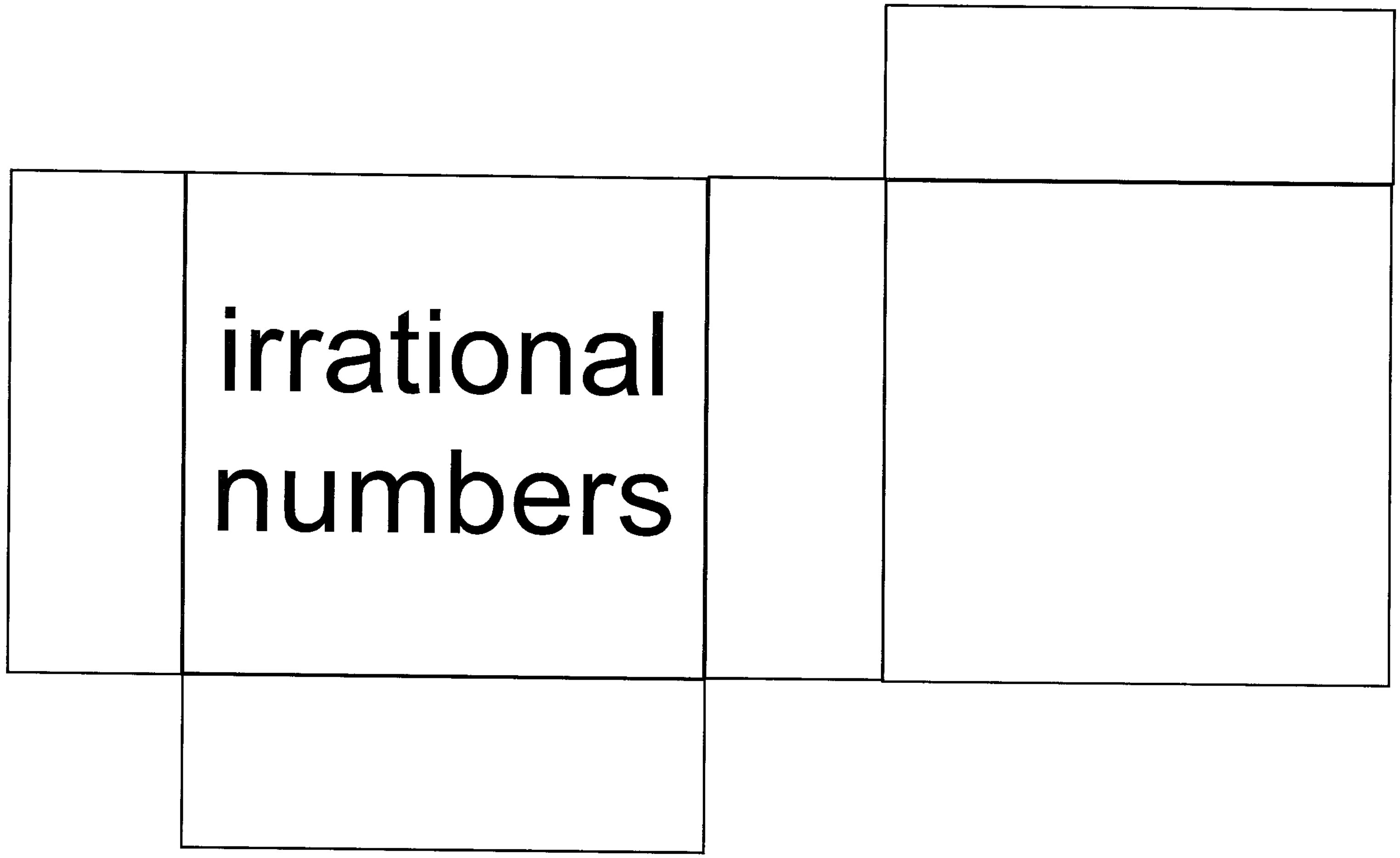
real numbers

rational
numbers

integers


whole
numbers


natural
numbers


irrational numbers