FARMER JOHN

- Percents
- Factors
- Spatial visualization
- Perimeters

Getting Ready

What You'll Need

Cuisenaire Rods, 1 set per group

1-centimeter grid paper, page 120

Activity Master, page 106

Overview

Students use Cuisenaire Rods and their given percentage values to determine possible whole number bases. Then they build models representing the given percentages in order to find the one with the smallest perimeter. In this activity, students have the opportunity to:

- determine common multiples for a set of numbers
- calculate percents of various bases
- represent percentages of a whole visually
- compute perimeters

Other *Super Source* activities that explore these and related concepts are:

Exercises on the Uneven Bars, page 44

Tangram Boomerang, page 53

House of Representatives, page 58

The Activity

On Their Own (Part 1)

Before planting the Spring crops, Farmer John needs to determine how many acres will be allocated for his vegetables. He decides to plant 5% of the area with onions, 20% with tomatoes, 25% with eggplant, 25% with corn, and 25% with carrots. The number of acres for each crop must be whole numbers. How can John determine the minimum number of acres that will be planted and the acreage allotted each vegetable?

- Work with a partner. Determine possible values for the number of acres John may need, given the percentage values for each crop mentioned above.
- John must also consider that different plants have different space requirements. Use Cuisenaire Rods to represent the required spaces as follows: onions—white rods, tomatoes—red rods, eggplant—purple rods, corn—yellow rods, and carrots—orange rods. Use as many of each color as you need.
- If the acreage to be planted must contain only complete "vegetable" rods and 1 unit rod is equal to the area of 1 acre, determine the minimum number of acres that John must plant.
- Be ready to explain the method(s) used to arrive at your solutions.

Thinking and Sharing

Invite a pair of students to share their solutions for the number of acres and discuss the method(s) they used to determine them.

Use prompts like these to promote class discussion:

- Did you choose to work with the percentages or did you change them into their equivalent fractions? Why?
- How did you go about searching for whole numbers that could serve as the base? What values did you discover?
- How did the length of the rod representing the space needed for each vegetable affect the size of the entire garden area needed?
- What was the minimum size of the garden needed to accommodate the entire set of crops?

On Their Own (Part 2)

What if... ***John must plan the layout of the garden for these five crops? If different vegetables must be grouped by type and the garden area must be rectangular in shape to facilitate irrigation, how can John arrange the garden?***

- Use the appropriate color and number of Cuisenaire Rods as determined in the first activity to represent each crop.
- Design a rectangular garden layout using the rods; be sure to group rods of the same color together, so that each rod of a given color touches at least one other rod of that color.
- Record the garden layout on centimeter grid paper, being sure to label the placement of each vegetable crop.
- Make, record, and label as many other rectangular gardens as possible.
- To save on the cost of irrigation pipes, John would like to use the garden whose dimensions yield the smallest perimeter. Determine which of your gardens has the smallest perimeter.
- Be ready to explain how you went about designing your garden layouts.

Thinking and Sharing

Invite students to display their garden layouts, stating the perimeter of each. Continue posting layouts until students feel they have a sufficient variety and can agree on the dimensions producing the minimum perimeter.

Use prompts like these to promote class discussion:

- How did you go about arranging the rods of the same color?
- How did you go about finding a rectangular layout that would use all the rods?

- How can you use one rectangular garden layout to generate others?
- What do each of the garden layouts have in common?
- Is it possible for two gardens to have the same areas and perimeters but be arranged differently? Explain.
- How did the perimeters of the various garden layouts compare to each other?
- Which rectangle produced the minimum perimeter?

For Their Portfolio

Write a brief letter to John explaining the method used to determine the number of acres needed in his garden to accommodate the appropriate percentage of each vegetable.

Teacher Talk

Where's the Mathematics?

Students may begin by randomly choosing whole numbers to use as the base for the percentage calculations. Starting with 100 acres, which is perhaps the most logical base to use when working with percents, students determine that the acre allotments would be 25 for eggplant, 25 for carrots, 25 for corn, 20 for tomatoes, and 5 for onions. But students must then determine if this represents the minimum number of acres needed.

Other students may begin by considering the smallest percentage, figuring that the 5% allotment will play a major role in determining the minimum number of acres. With this idea in mind, students may conclude that 20 acres is the smallest total for which 5% is a whole number (because 5% = $\frac{1}{20}$). It then becomes necessary to determine if the larger percents calculated on a base of 20 will also yield whole numbers, which in this case, they do.

Another approach involves converting the percentages to their equivalent fractions, as follows: 25% = $\frac{1}{4}$, 20% = $\frac{1}{5}$ and 5% = $\frac{1}{20}$. Knowing that these fractional parts of the entire garden must end up as whole numbers, students may choose to find the lowest common denominator, 20, for the three fractions. Students may conclude that this is the minimum number of acres needed for the garden.

Students must remember that if only complete rods can be used, the length of each differently colored rod also affects the garden area required to satisfy the percentages. The five crops use rods whose lengths are as follows: onions—1 unit, tomatoes—2 units, eggplants—4 units, corn—5 units, and carrots—10 units. If, for example, the entire garden area is 20 acres, the number of acres allocated for carrots would be 25%, or 5 acres. However, the length of a carrot rod is 10 units and can appear only in segments of 10, so they could occupy a total of 10, 20, or 30 units, and so on. Students can therefore conclude that 20 acres will not work as the minimum number of acres for the garden's area, because a single carrot rod would take up 50% of the 20 acres.

Also, since onions occupy $\frac{1}{20}$ of the garden's area, and each crop must be in whole acres, students may realize that the actual total acreage of the garden must be a multiple of 20 acres.

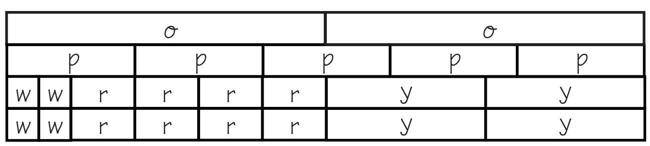
Students can continue to check multiples of the 20 acres to find the smallest multiple that will yield whole numbers for the lengths and percentages of the various vegetable rods. Creating charts similar to the one shown, may help students organize their findings.

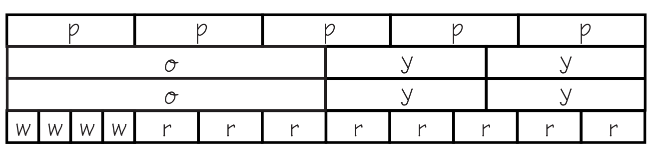
	5% onions (1 unit each)	20% tomatoes (2 units each)	25% eggplants (4 units each)	25% corn (5 units each)	25% carrots (10 units each)
20-acre garden	1 rod = 1 acre	2 rods = 4 acres	$1\frac{1}{4}$ rods = 5 acres	1 rod = 5 acres	$\frac{1}{2}$ rod = 5 acres
40-acre garden	2 rods = 2 acres	4 rods = 8 acres	$2\frac{1}{2}$ rods = 10 acres	2 rods = 10 acres	1 rod = 10 acres
60-acre garden	3 rods = 3 acres	6 rods = 12 acres	$3\frac{3}{4}$ rods = 15 acres	3 rods = 15 acres	$1\frac{1}{2}$ rods = 15 acres
80-acre garden	**4 rods = 4 acres**	**8 rods = 16 acres**	**5 rods = 20 acres**	**4 rods = 20 acres**	**2 rods = 20 acres**

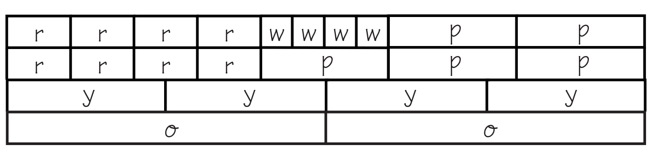
The circled fractions in the chart above eliminate the 20-, 40-, and 60- acre gardens as possible solutions. Students should conclude that 80 acres is the smallest multiple of 20 for which the percents of the garden are whole numbers and the number of "vegetable" rods representing each crop is also a whole number. Each of the lengths of the vegetable rods must, therefore, be a factor of the number of acres allocated for that crop to insure the required whole number result.

In planning the garden layout in Part 2, students may start by arranging the rods for each type of vegetable together and then try attaching these groupings to form one large rectangle.

New layouts can then be generated from existing layouts by moving sections as shown below.







Or students may frame out a rectangle whose area is 80 units and try arranging the vegetable rods within this frame. Since the dimensions of any of these rectangles must also be whole numbers, students can determine that the garden dimensions could be only 1x80, 2x40, 4x20, 5x16, or 8x10. Students may decide before trying different rod layouts to calculate the perimeters of the five rectangles just identified to limit the layout possibilities. These perimeters are, respectively: 162, 84, 48, 42, and 36 units. Therefore, the garden whose area is 80 acres must measure 8x10 in order to produce the minimum perimeter. Some possible layouts are shown below.