SQUARES AND STAIRCASES

NUMBER • PATTERNS/FUNCTIONS

- Square numbers
- Pattern recognition
- Growth patterns

Getting Ready

What You'll Need

Snap Cubes, 50–60 per pair

Calculators, 1 per pair

Snap Cube grid paper, page 91 (optional)

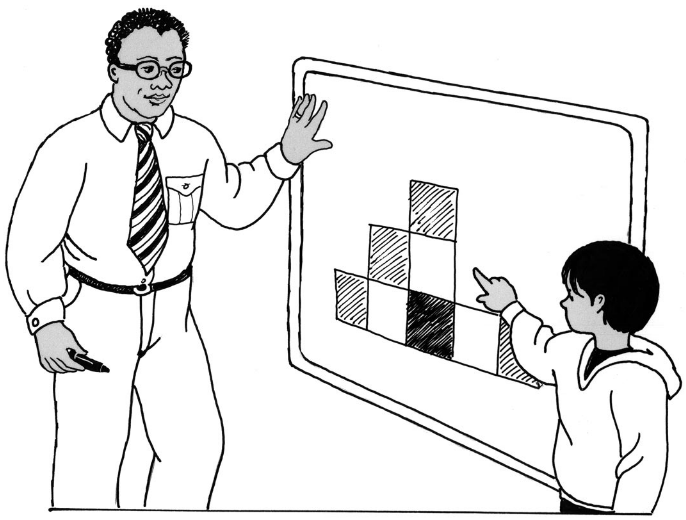
Crayons (optional)

Overhead Snap Cubes and/or Snap Cube grid paper transparency (optional)

Overview

Children use Snap Cubes to build a non-typical model of square numbers. They record data, look for patterns, and make conjectures. In this activity, children have the opportunity to:

- represent a numerical sequence geometrically
- collect and analyze data
- learn about a predictable growth pattern
- use patterns to make predictions

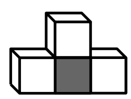


The Activity

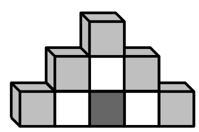
Use any combination of colors. The different colors help children focus on the change in the number of cubes from one staircase to the next.

Introducing

- Display a red Snap Cube. Identify the six faces of the cube: the top, bottom, right, left, front, and back.
- Tell children that you would like to build a staircase by adding a white cube to the top, left, and right faces. Ask how many white cubes you will need. Add the white cubes and count to confirm that the new staircase contains four cubes.



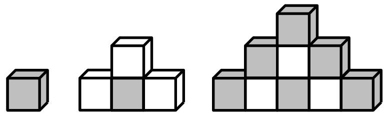
- Ask children what the staircase would look like if you again added cubes—this time blue ones—to the top, left, and right faces of the four-cube staircase.
- Have children share their predictions, then add the blue cubes.
- Display the new staircase, which now has a total of nine cubes.



On Their Own

Is there a way to know in advance the number of Snap Cubes you would need and how to connect them in order to build a staircase of any size?

- Work with a partner to build staircases that look like these:



- Keep track of the number of cubes you add each time, the total number of cubes in each staircase, and the patterns you find.
- Predict what the 4th staircase will look like, then build it.
- Build staircases until you can use your findings to describe, in detail, the 100th staircase without building it.

The Bigger Picture

Thinking and Sharing

Create a class chart that has three columns. Label the first column *Staircase Number,* the second column *Number of Cubes Added,* and the third column, *Total Number of Cubes.* Have children fill in the chart and discuss the data.

Use prompts such as these to promote class discussion:

- What did you notice as you built bigger and bigger staircases?
- What patterns do you see in the data?
- How many structures did you need to build before you could make predictions about the 100th staircase?
- How did you find the number of cubes needed for the 100th staircase?
- How is the 100th staircase different from the 99th staircase? from the 101st staircase?
- Mathematicians call the number sequence generated by the total number of cubes *square numbers.* Why do you think they are called square numbers?

Extending the Activity

1. Have children graph their data. On one graph, the numbers along the horizontal axis can represent the "Staircase Number" and the numbers along the vertical axis can represent the "Number of Cubes Added." On a second graph, have children change the vertical axis so it represents the "Total Number of Cubes." Have children compare their graphs.
2. Have children describe, in detail, a staircase whose bottom step contains 41 cubes.

Teacher Talk

Where's the Mathematics?

This activity provides a fresh approach to looking at square numbers. Generally, the sequence is studied by building larger and larger cubes and focusing on their square faces. Building square numbers in a new fashion helps children to become more open and flexible in their thinking. This activity provides children with an opportunity to use mental math as they analyze the patterns they find.

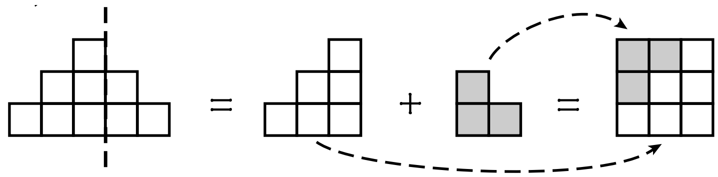
The class chart should look like this.

Staircase Number	Number of Cubes Added	Total Number of Cubes
1	0	1
2	3	4
3	5	9
4	7	16
5	9	25
6	11	36
7	13	49
8	15	64
9	17	81
10	19	100

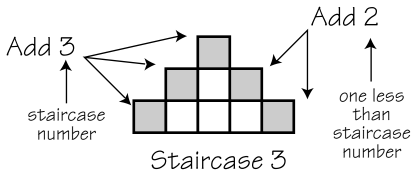
Looking at their structures, children can notice many patterns. To begin, every staircase number has a matching number of stairsteps, or layers. That is, staircase 1 has one layer, staircase 2 has two layers, and so on. Every staircase has one cube at the top, and each layer has an odd number of cubes. In addition, all the staircases have symmetry. A vertical line of symmetry goes through the tallest column of cubes in each staircase.

There is more than one way to build a staircase. Some children add one cube to each exposed face on the left, right, and top. Others add a bottom layer that has two more cubes than the bottom layer of the previous staircase. This means that the number of cubes added is always the same as the number of cubes in the bottom layer.

When asked to describe the numbers in the "Total Number of Cubes" column, children usually recognize that each number is the staircase number multiplied by itself. Identify these numbers as *square numbers.* Encourage children to explain why a staircase that looks roughly like a triangle can show square numbers. If no one can provide an explanation, give a hint by splitting a staircase into two pieces, as shown below. Suggest to children that they rotate and move the two pieces to form a square.



Children will describe the “Number of Cubes Added” column in a variety of ways. The most obvious will be the sequence of odd numbers. Others will describe it as the new “total number of cubes” minus the previous “total number of cubes,” that is, 3 = 4 – 1, 5 = 9 – 4, 7 = 16 – 9, and so on. Others may say that it is the staircase number plus the staircase number less one, that is, 3 = 2 + 1, 5 = 3 + 2, 7 = 4 + 3, and so on. These children might point this out on the staircase itself.



Once they recognize and describe the growth patterns they find in their staircases and their data, children can describe any size staircase. For example, staircase 8 has 8 layers and 64 cubes. This is so since 8 x 8 = 64 and 1 + 3 + 5 + 7 + 11 + 13 + 15 = 64. The 100th staircase is 100 layers high and contains 100 x 100 or 10,000 cubes. It has 199 more cubes that the 99th staircase.

Encourage children to think in a different way. Tell them how many cubes are in the bottom layer of a staircase, then ask them to predict how many layers, or stairsteps, are in the staircase. To illustrate, imagine a staircase with a base layer of 41 cubes. Some children will work backwards, using the pattern they found to describe the “number of cubes added.” For example, if they realize that the “number of cubes added” is twice the staircase number minus one, they may think “If two times a number minus 1 is 41, then two times that number is 42, which means that the number must be 21.” Other children may extend their chart until they get to 41. No matter the method, children should find that a staircase with a bottom layer of 41 is the 21st staircase and has a total 21 x 21 or 441 cubes.