Farkle: A Game of Probability & Chance

What is Farkle? Farkle is the name of a game played with dice. It has similarities to Greed and Yahtzee. It has several versions of the rules, one of which I will present today. Once the basics are understood, the rules may be adapted for different situations.

What do you need? Six fair six-sided dice
A sheet of paper and a pencil

How many people can play? There is no limit but at least 2. The more people in the game, the longer it takes to play. Between 2 and 4 people playing is best if the game must be finished within 50 minutes.

What is the objective? The first person to 10,000 points wins the game. You may adjust the point limit to 5,000 or 8,000 points if you need to shorten the game.

How do you play? Roll all 6 dice. Certain combinations of numbers earn points as follows:
1 alone = 100 points
5 alone = 50 points
1, 1, 1 = 1,000 points
1, 2, 3, 4, 5, 6 = 1,000 points
3 pairs = 1,000 points (Ex. 2, 2, 5, 5, 1, 1)
3 of a kind = (number rolled)×100 (Ex. 4, 4, 4 = 400 points, except for 1, 1, 1)

After the first roll, decide to either take the points from the 1st roll, and record them on the sheet of paper under your name, or remove at least one die or set of dice that score points and continue rolling with the remaining dice.

If you roll and have no dice that score points, you lose your turn and all points accumulated on the 1st roll.

If on your second roll, you score more points, add them to the points scored on the first roll and then decide again whether to stop or roll again. If you stop, you keep all points accumulated. If you roll again, there's a chance that all points will be lost. And, so on…

If all 6 dice are scored upon in one turn, then you have "Farkled," and can pick up all 6 dice, continue rolling, and continue adding points to your total. Again, if you roll no scoring dice, you lose all accumulated points.

Optional Rules: You must score at least 500 points initially to "get on the scoreboard."

Once a player equals or exceeds 10,000 points, all other players get one additional turn to try and pass up the leader.

Probability Questions: Playing Farkle can be a good way to review probability with students for the TAKS tests. The following questions relate to the game and are sequenced to lead students to higher levels of thinking. Some more advanced questions are near the end. Answers are included, and you may modify these questions to achieve your desired end.

1. Situation: Five dice have been scored on, and you have one die left to roll.
   a) What's the probability that you roll a 1?
   $\frac{1}{6}$
   b) What's the probability that you roll a 5?
   $\frac{1}{6}$
   c) What's the probability that you roll something other than 5 or 1?
   $\frac{4}{6} = \frac{2}{3}$
   d) What's the probability that you "farkle?"
   $\frac{2}{6} = \frac{1}{3}$
   e) If you roll, do you expect to see a 1 or 5, or do you expect to see a 2, 3, 4, or 6? Explain why.
   You would expect to see a 2,3,4, or 6 since you have 4 chances at one of those numbers and only 2 chances for a 1 or 5.

   f) Do you know for sure that you won't roll a 1 or 5?
   No. It is possible, but it is not the most probable outcome.

g) If you're in this situation on 6 different occasions, how many times do you expect to farkle?
Twice.

h) Are you assured that you'll farkle on any of those 6 rolls?
No, but in the long run, about twice out of every 6 rolls will be a farkle.

i) If you're in this situation, would you roll the die or stay put? Explain your reasoning behind your strategy.
Answers will vary, but my strategy is to roll if I'm behind and stay put if I'm ahead.

2. Situation: Four dice have been scored on, and you have 2 dice left to roll.

a) What's the probability that exactly one 1 is rolled and no 5's?
$\frac{8}{36} = \frac{2}{9}$

b) What's the probability that exactly one 5 is rolled and no 1's?
$\frac{8}{36} = \frac{2}{9}$

c) What's the probability of rolling a 1 and a 5?
$\frac{2}{36} = \frac{1}{18}$

d) What's the probability of rolling double 1's or double 5's?
$\frac{2}{36} = \frac{1}{18}$

e) What's the probability that you do not lose all your points on the roll?
$\frac{8}{36} + \frac{8}{36} + \frac{2}{36} + \frac{2}{36} = \frac{20}{36} = \frac{5}{9}$

f) How does this probability compare to when you have only one dice to roll? (See question #1 (d).) Explain why this is.
It is the higher. With more dice to roll, there is a smaller probability of losing your points.

More Advanced Questions:

3. a) What's the probability of rolling a 1, 2, 3, 4, 5, 6 (a straight)?

   $$(6!)\times\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{720}{46{,}656}=\frac{5}{324}\approx 0.015$$

   b) Is this event likely to happen?

   Pretty unlikely

   c) In 1,000 rolls, about how many times would this sequence occur (on average)?

   15

4. a) What's the probability of rolling exactly three ones at a time?

   $$\binom{6}{3}\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=\frac{2500}{46{,}656}=\frac{625}{11664}\approx 0.054$$

   b) Are you more or less likely to roll exactly three ones or a straight?

   More likely to roll exactly three ones

5. a) What's the maximum points possible on one roll of all 6 dice?

   2,000, by rolling all ones

   b) What is the probability that this particular roll occurs?

   $$\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{46{,}656}\approx 0.00002$$

6. a) What is the probability of rolling exactly 3 of a kind (excluding ones)?

   $$5\times\binom{6}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3=0.2679$$

   b) What is the probability of rolling at least 3 of a kind (excluding ones)?

   $$5\times\sum_{i=3}^{6}\binom{6}{i}\left(\frac{1}{6}\right)^i\left(\frac{5}{6}\right)^{6-i}=5\times(0.623)=0.3114$$

7. a) What's the probability of rolling three pairs?

   $$\binom{6}{3}\times\binom{6}{3}\times\left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\right)=\frac{400}{46{,}656}=\frac{25}{2{,}916}\approx 0.0086$$

   b) Is this probability greater or less than that for rolling a straight?

   Less.

Real-life Discussion: There are risks and decisions everywhere in life. Ideally, we would all make the right choices, but there are no guarantees. We each need to learn how to weigh the advantages and disadvantages of a given choice for our individual circumstances.

Example 1: You have two ways in which you can invest a sum of money. Each has a different amount of risk of losing all of your money, and each has a different interest rate.

1. Risk: 10% Rate: 5%
2. Risk: 60% Rate: 25%

Which would you choose, and why?

Example 2: Alternatively, you may earn a higher interest rate when you commit your money to be invested for a different length of time. (Penalties apply to withdrawing it early.) Which of the following two choices would you pick?

1. Length of Time: 5 years Rate: 7%
2. Length of Time: 6 months Rate: 3%